Buckling of Quasisinusoidally Corrugated Plates in Shear

Susumu Toda*
National Aerospace Laboratory, Tokyo, Japan
and
Shigeo Sanbongi†
National Space Development Agency of Japan, Tokyo, Japan

This paper determines the buckling stress of quasisinusoidally corrugated plates in shear. Governing equations are obtained by the variations of potential energy, which is evaluated by assuming the buckling deflection components in the form of a Fourier series. An approximate but closed-form solution is then derived to determine the governing parameters of the problem. The present analytical results are compared favorably with both finite element methods and experimental results.

Introduction

THE corrugated plate has usually been replaced in theoretical buckling analysis by an equivalent flat plate having orthotropic material properties. In order to assess the accuracy of this approach, refined numerical studies of the buckling of corrugated plates have been undertaken by several investigators. For example, Libove and his colleague have been studying the shear buckling loads for both quasi-sinusoidally and trapezoidally corrugated plates, their analysis being based on the principle of stationary potential energy.[1,2,8] They have shown that orthotropic plate theory can seriously underestimate the shear buckling load. Sanbongi has conducted a parametric finite element method (FEM) study using high-precision finite strip elements based on Sanders shell theory and has presented the results on the buckling of circularly corrugated plates under axial compression[3] and shear.[4] In order to give an analytical verification for the numerical results in Ref. 3, Toda has presented an analytical solution to buckling of sinusoidally corrugated plates under axial compression.[5]

The purpose of the present paper is to give an analytical explanation of the local buckling characteristics of quasi-sinusoidally corrugated plates in shear, the overall buckling of which has been studied in detail by Libove and Hussain.[8] Assuming the buckling deflection components in the form of a Fourier series, the second variation of the potential energy is evaluated with the aid of a Donnell-type approximation. Then the eigenvalue equations for neutral equilibrium are obtained by variations in the potential energy increment. Finally, an approximate but closed-form solution is presented for the critical shearing stress and is compared with both finite element and experimental results.

Theoretical Analysis

Let us consider a thin sinusoidally corrugated plate of isotropic material with Young's modulus E and Poisson's ratio ν. The coordinate system (x,s) is defined in the middle surface, such that x measures the axial distance along the generator and s is the arc length along the corrugation. The third coordinate z is taken in the normal direction. During the buckling of the plate in uniform shear stress τ, the deflection components in these directions are denoted u, v, and w, respectively. The geometry and the coordinate system of the analytical model for a corrugated plate with thickness t is shown in Fig. 1, where the local radius of curvature $\rho(s)$ is defined by

$$1/\rho = (1/R)\cos(\pi s/b) \tag{1}$$

where R is the radius of curvature at the crests. The following geometric relations are easily derived:

$$\begin{aligned} \frac{\pi p}{b} &= \int_0^{\pi} \cos(\kappa \sin\xi)\,d\xi \equiv f_1 \\ \frac{\pi h}{\kappa b} &= \frac{1}{\kappa}\int_0^{\pi} \sin(\kappa\sin\xi)\,d\xi \equiv f_2 \end{aligned} \tag{2}$$

with

$$\kappa = b/\pi R \tag{3}$$

The values of f_1, f_2, and h/p are plotted against κ in Fig. 2.

The second variation of the potential energy by transition from the unbuckled fundamental state to the buckled configuration at the same load is written in the form

$$P_2 = U_m + U_b + U_w \tag{4}$$

where

$$U_m = \frac{Et}{2(1-\nu^2)}\int_0^a\int_0^\ell \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial s}+\frac{w}{\rho}\right)^2 + 2\nu\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial s}+\frac{w}{\rho}\right) + \frac{1-\nu}{2}\left(\frac{\partial u}{\partial s}+\frac{\partial v}{\partial x}\right)^2\right] dx\,ds \tag{5a}$$

$$U_b = \frac{Et^3}{24(1-\nu^2)}\int_0^a\int_0^\ell \left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial s^2}\right)^2 + 2\nu\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial s^2}\right) + 2(1-\nu)\left(\frac{\partial^2 w}{\partial x\partial s}\right)^2\right] dx\,ds \tag{5b}$$

$$U_w = \tau t\int_0^a\int_0^\ell \frac{\partial w}{\partial x}\,\frac{\partial w}{\partial s}\,dx\,ds \tag{5c}$$

Let us assume a displacement field during buckling as

$$u = \sum_m \sum_n U_{mn} \sin\frac{n\pi s}{\ell}\cos\frac{m\pi x}{a} \tag{6a}$$

Received Oct. 8, 1984; revision received April 9, 1985.

*Head, Structure Laboratory; presently on assignment with the National Space Development Agency of Japan. Member AIAA.
†Senior Engineer, First Satellite Design Group.

$$v = \sum_m \sum_n V_{mn} \cos\frac{n\pi s}{\ell} \sin\frac{m\pi x}{a} \tag{6b}$$

$$w = \sum_m \sum_n W_{mn} \sin\frac{n\pi s}{\ell} \sin\frac{m\pi x}{a} \tag{6c}$$

where

$$\ell = Nb, \quad (N=1,2,3,\ldots) \tag{6d}$$

is a buckling half-wavelength. These deflections indentically satisfy the boundary condition specified by

$$w = \frac{\partial^2 w}{\partial x^2} = N_x = v = 0 \text{ at } x=0 \text{ and } x=a \tag{7}$$

where N_x denotes the axial stress resultant.

Evaluating the integrals in Eqs. (5) and taking the variation of P_2 with respect to U_{mn}, V_{mn}, and W_{mn}, we obtain the set of governing equations as follows:

$$\nu\phi_m \sum_{q=1} A_{qn} W_{mq} + \left(\phi_m^2 + \frac{1-\nu}{2}\psi_n^2\right) U_{mn} + \frac{1+\nu}{2}\phi_m\psi_n V_{mn} = 0 \quad (m,n=1,2,3,\ldots) \tag{8}$$

$$\psi_n \sum_{q=1} A_{qn} W_{mq} + \frac{1+\nu}{2}\phi_m\psi_n U_{mn} + \left(\frac{1-\nu}{2}\phi_m^2 + \psi_n^2\right) V_{mn} = 0 \quad (m,n=1,2,3,\ldots) \tag{9}$$

$$\alpha W_{mn}(\phi_m^2+\psi_n^2)^2 + \sum_{q=1} B_{nq} W_{mq} + \nu\phi_m \sum_{q=1} A_{nq} U_{mq} + \sum_{q=1} \psi_q A_{nq} V_{mq} + 4\lambda \sum_{p=1} \sum_{q=1} (\phi_m\psi_q P_{qn} P_{mp} + \phi_p\psi_n P_{nq} P_{pm}) W_{pq} = 0 \quad (m,n=1,2,3,\ldots) \tag{10}$$

where

$$\phi_m = \frac{m\pi R}{a}, \quad \psi_n = \frac{n\pi R}{\ell}, \quad \alpha = \frac{t^2}{12R^2}, \quad \lambda = \frac{\tau(1-\nu^2)}{E} \tag{11}$$

Furthermore,

$$\begin{aligned} P_{mp} &= \frac{1}{a}\int_0^a \cos\frac{m\pi x}{a}\sin\frac{p\pi x}{a}dx \\ &= 0 \qquad \text{if } p \pm m = \text{even} \\ &= \frac{2}{\pi}\frac{p}{p^2-m^2} \qquad \text{if } p \pm m = \text{odd} \end{aligned} \tag{12}$$

$$\begin{aligned} A_{nq} = A_{qn} &= -\frac{2}{\ell}\int_0^\ell \cos\frac{\pi s}{b}\sin\frac{n\pi s}{\ell}\sin\frac{q\pi s}{\ell}ds \\ &= -\tfrac{1}{2} \qquad \text{if } (n-q) = N \\ &= \tfrac{1}{2} \qquad \text{if } (n+q) = N \\ &= 0 \qquad \text{if } (n \pm q) \neq N \end{aligned} \tag{13}$$

Table 1 Values of A_{nq} and B_{nq}

$A_{nq} = A_{qn}$

$N=1$

n \ q	1	2	3	4	5	6	7	…
1	0	−½	0	0	0	0	0	
2	−½	0	−½	0	0	0	0	
3	0	−½	0	−½	0	0	0	
4	0	0	−½	0	−½	0	0	
5	0	0	0	−½	0	−½	0	
6	0	0	0	0	−½	0	−½	
7	0	0	0	0	0	−½	0	
⋮								

$N=2$

n \ q	1	2	3	4	5	6	7	…
1	½	0	−½	0	0	0	0	
2	0	0	0	−½	0	0	0	
3	−½	0	0	0	−½	0	0	
4	0	−½	0	0	0	−½	0	
5	0	0	−½	0	0	0	−½	
6	0	0	0	−½	0	0	0	
7	0	0	0	0	−½	0	0	
⋮								

$B_{nq} = B_{qn}$

$N=1$

n \ q	1	2	3	4	5	6	7	…
1	¼	0	¼	0	0	0	0	
2	0	½	0	¼	0	0	0	
3	¼	0	½	0	¼	0	0	
4	0	¼	0	½	0	¼	0	
5	0	0	¼	0	½	0	¼	
6	0	0	0	¼	0	½	0	
7	0	0	0	0	¼	0	½	
⋮								

$N=2$

n \ q	1	2	3	4	5	6	7	…
1	½	0	−¼	0	¼	0	0	
2	0	¼	0	0	0	¼	0	
3	−¼	0	½	0	0	0	¼	
4	0	0	0	½	0	0	0	
5	¼	0	0	0	½	0	0	
6	0	¼	0	0	0	½	0	
7	0	0	¼	0	0	0	½	
⋮								

and

$$B_{nq}=B_{qn}=\frac{2}{\ell}\int_0^\ell \cos^2\frac{\pi s}{b}\sin\frac{n\pi s}{\ell}\sin\frac{q\pi s}{\ell}ds$$
$$=1/4 \quad \text{if } q=n=N$$
$$=1/2 \quad \text{if } q=n\neq N$$
$$=1/4 \quad \text{if } (n-q)=2N$$
$$=-1/4 \quad \text{if } (n+q)=2N \tag{14}$$

Thus, the values of A_{nq} and B_{nq} are dependent on the value of N and are shown in Table 1 for $N=1$ and 2.

Solving Eqs. (8) and (9) for U_{mn} and V_{mn}, we obtain

$$U_{mn}=\frac{\phi_m(\nu\phi_m^2-\psi_n^2)}{2(\phi_m^2+\psi_n^2)^2}(W_{m,n-N}+W_{m,n+N}) \quad (m,n=1,2,3,\ldots)$$

$$V_{mn}=\frac{\psi_n[(2+\nu)\phi_m^2+\psi_n^2]}{2(\phi_m^2+\psi_n^2)^2}(W_{m,n-N}+W_{m,n+N}) \quad (m,n=1,2,3,\ldots) \tag{15}$$

where if $n-N<0$, $W_{m,n-N}=-W_{m,N-n}$ and if $n-N=0$, $W_{m,n-N}=0$.

Equations (15) show that U_{mn} and V_{mn} are expressed in terms of $W_{m,n-N}$ and $W_{m,n+N}$. Substitution of Eqs. (15) into Eq. (10) yields

$$\alpha W_{mn}(\phi_m^2+\psi_n^2)^2+\sum_{q=1} B_{nq}W_{mq}$$
$$+\sum_{q=1} A_{nq}\left[\frac{1}{2}-\frac{(1-\nu^2)\phi_m^4}{2(\phi_m^2+\psi_q^2)^2}\right](W_{m,q-N}+W_{m,q+N})$$
$$+4\lambda\sum_{p=1}\sum_{q=1}(\phi_m\psi_q P_{qn}P_{mp}+\phi_p\psi_n P_{nq}P_{pm})W_{pq}=0 \quad (m,n=1,2,3,\ldots) \tag{16}$$

Equations (16) are divided into two groups, one containing constants W_{mn} for which $m+n$ are even numbers (case I) and the other for which $m+n$ are odd numbers (case II). The buckling shear stress τ_{cr} is determined by the smallest values of λ for which the above two systems of equations have a nontrivial solution. Let us derive here the solutions for τ_{cr} in the cases where $N=1$ and 2.

$N=1$

Case I: $m+n=even$

Using Eq. (12) and Table 1, we can write the first five equations of Eqs. (16) for which $m+n$ are even numbers in the following form:

$$\left[\frac{(1-\nu^2)\psi_1^4}{4(\phi_1^2+\psi_2^2)^2}+\alpha(\phi_1^2+\psi_1^2)^2\right]W_{11}+\frac{(1-\nu^2)\phi_1^4}{4(\phi_1^2+\psi_2^2)^2}W_{13}-4\lambda\frac{32R^2}{9a\ell}W_{22}=0$$

$$\frac{(1-\nu^2)\phi_1^4}{4(\phi_1^2+\psi_2^2)^2}W_{11}+\left[\frac{(1-\nu^2)\phi_1^4}{4(\phi_1^2+\psi_2^2)^2}+\frac{(1-\nu^2)\phi_1^4}{4(\phi_1^2+\psi_4^2)^2}+\alpha(\phi_1^2+\psi_3^2)^2\right]W_{13}+4\lambda\frac{32R^2}{5a\ell}W_{22}=0$$

$$\left[\frac{(1-\nu^2)\phi_3^4}{4(\phi_3^2+\psi_2^2)^2}+\alpha(\phi_3^2+\psi_1^2)^2\right]W_{31}+\frac{(1-\nu^2)\phi_3^4}{4(\phi_3^2+\psi_2^2)^2}W_{33}+4\lambda\frac{32R^2}{5a\ell}W_{22}=0$$

$$\frac{(1-\nu^2)\phi_3^4}{4(\phi_3^2+\psi_2^2)^2}W_{31}+\left[\frac{(1-\nu^2)\phi_3^4}{4(\phi_3^2+\psi_2^2)^2}+\frac{(1-\nu^2)\phi_3^4}{4(\phi_3^2+\psi_4^2)^2}+\alpha(\phi_3^2+\psi_3^2)^2\right]W_{33}-4\lambda\frac{288R^2}{25a\ell}W_{22}=0$$

$$-4\lambda\frac{32R^2}{9a\ell}W_{11}+4\lambda\frac{32R^2}{5a\ell}W_{13}+4\lambda\frac{32R^2}{5a\ell}W_{31}-4\lambda\frac{288R^2}{25a\ell}W_{33}+\left[\frac{(1-\nu^2)\phi_2^4}{4(\phi_2^2+\psi_1^2)^2}+\frac{(1-\nu^2)\phi_2^4}{4(\phi_2^2+\psi_3^2)^2}+\alpha(\phi_2^2+\psi_2^2)^2\right]W_{22}=0$$

Equating to zero the determinant of these equations, the smallest eigenvalue is obtained as

$$\lambda^{*2}=T/S \tag{17}$$

where

$$\lambda^*=4\lambda\frac{32R^2}{a\ell}=\frac{128(1-\nu^2)R^2\tau}{a\ell E} \tag{18}$$

$$T=A_{22} \tag{19}$$

$$S=\frac{A_{11}}{9F_2(\beta)}+\frac{A_{13}}{5F_2(\beta)}+\frac{A_{31}}{5F_2(3\beta)}+\frac{9A_{33}}{25F_2(3\beta)} \tag{20}$$

$$A_{22}=\frac{4(1-\nu^2)\beta^4}{(1+4\beta^2)^2}+\frac{4(1-\nu^2)\beta^4}{(9+4\beta^2)^2}+\frac{16\alpha}{\kappa^4}(1+\beta^2)^2$$

$$A_{11}=\frac{7(1-\nu^2)\beta^4}{90(4+\beta^2)^2}+\frac{(1-\nu^2)\beta^4}{36(16+\beta^2)^2}+\frac{\alpha}{9\kappa^4}(9+\beta^2)^2$$

$$A_{13}=\frac{7(1-\nu^2)\beta^4}{90(4+\beta^2)^2}+\frac{\alpha}{5\kappa^4}(1+\beta^2)^2$$

$$A_{31}=\frac{567(1-\nu^2)\beta^4}{50(4+9\beta^2)^2}+\frac{81(1-\nu^2)\beta^4}{20(16+9\beta^2)^2}+\frac{81\alpha}{5\kappa^4}(1+\beta^2)^2$$

$$A_{33}=\frac{567(1-\nu^2)\beta^4}{50(4+9\beta^2)^2}+\frac{9\alpha}{25\kappa^4}(1+9\beta^2)^2$$

$$F_2(\beta)=\frac{(1-\nu^2)^2\beta^8}{16(4+\beta^2)^2(16+\beta^2)^2}+\frac{\alpha(1-\nu^2)\beta^4}{4\kappa^4}\times\left[\left(\frac{1+\beta^2}{4+\beta^2}\right)^2+\left(\frac{9+\beta^2}{4+\beta^2}\right)^2+\left(\frac{1+\beta^2}{16+\beta^2}\right)^2\right]+\frac{\alpha^2}{\kappa^8}(1+\beta^2)^2(9+\beta^2)^2 \tag{21}$$

with

$$\beta=\ell/a$$

In the limiting case of $R\to\infty$, Eq. (17) reduces to

$$\left[\frac{\pi^4 D}{32\beta a^2 t\tau}\right]^2=\frac{\beta^4}{81(1+\beta^2)^4}\left[1+\frac{81}{625}+\frac{81}{25}\left(\frac{1+\beta^2}{9+\beta^2}\right)^2+\frac{81}{25}\left(\frac{1+\beta^2}{1+9\beta^2}\right)^2\right] \tag{22}$$

This is identical to the critical shearing stress for a flat plate given in the textbook by Timoshenko and Gere.[6]

Case II: $m+n=odd$

By taking four equations with coefficients W_{12}, W_{21}, W_{23}, and W_{32} of Eqs. (16), we obtain

$$\left[\frac{(1-\nu^2)\phi_1^4}{4(\phi_1^2+\psi_1^2)^2}+\frac{(1-\nu^2)\phi_1^4}{4(\phi_1^2+\psi_3^2)^2}+\alpha(\phi_1^2+\psi_2^2)^2\right]W_{12}$$

$$+4\lambda\frac{32R^2}{9a\ell}W_{21}-4\lambda\frac{32R^2}{5a\ell}W_{23}=0$$

$$4\lambda\frac{32R^2}{9a\ell}W_{12}+\left[\frac{(1-\nu^2)\phi_2^4}{4(\phi_2^2+\psi_2^2)^2}+\alpha(\phi_2^2+\psi_1^2)^2\right]W_{21}$$

$$+\frac{(1-\nu^2)\phi_2^4}{4(\phi_2^2+\psi_2^2)^2}W_{23}-4\lambda\frac{32R^2}{5a\ell}W_{32}=0$$

$$-4\lambda\frac{32R^2}{5a\ell}W_{12}+\frac{(1-\nu^2)\phi_2^4}{4(\phi_2^2+\psi_2^2)^2}W_{21}+\left[\frac{(1-\nu^2)\phi_2^4}{4(\phi_2^2+\psi_2^2)^2}\right.$$

$$\left.+\frac{(1-\nu^2)\phi_2^4}{4(\phi_2^2+\psi_4^2)^2}+\alpha(\phi_2^2+\psi_3^2)^2\right]W_{23}+4\lambda\frac{288R^2}{25a\ell}W_{32}=0$$

$$-4\lambda\frac{32R^2}{5a\ell}W_{21}+4\lambda\frac{288R^2}{25a\ell}W_{23}+\left[\frac{(1-\nu^2)\phi_3^4}{4(\phi_3^2+\psi_1^2)^2}\right.$$

$$\left.+\frac{(1-\nu^2)\phi_3^4}{4(\phi_3^2+\psi_3^2)^2}+\alpha(\phi_3^2+\psi_2^2)^2\right]W_{32}=0$$

The critical stress is obtained by equating to zero the determinant of the above homogeneous equations, which is also expressed by

$$\lambda^{*2}=T/S$$

where

$$T=A_{12}A_{32} \tag{23}$$

$$S=\frac{1}{5F_1(\beta)}(5A_{21}+9A_{23})\left(\frac{A_{12}}{25}+\frac{A_{32}}{81}\right) \tag{24}$$

and

$$A_{12}=\frac{(1-\nu^2)\beta^4}{4(1+\beta^2)^2}+\frac{(1-\nu^2)\beta^4}{4(9+\beta^2)^2}+\frac{\alpha}{\kappa^4}(4+\beta^2)^2$$

$$\mathrm{A}_{21}=\frac{7(1-\nu^2)\beta^4}{10(1+\beta^2)^2}+\frac{(1-\nu^2)\beta^4}{4(4+\beta^2)^2}+\frac{\alpha}{\kappa^4}(9+4\beta^2)^2$$

$$A_{23}=\frac{7(1-\nu^2)\beta^4}{10(1+\beta^2)^2}+\frac{9\alpha}{5\kappa^4}(1+4\beta^2)^2$$

$$A_{32}=\frac{81(1-\nu^2)\beta^4}{4(1+9\beta^2)^2}+\frac{(1-\nu^2)\beta^4}{4(1+\beta^2)^2}+\frac{\alpha}{\kappa^4}(4+9\beta^2)^2$$

$$F_1(\beta)=\frac{(1-\nu^2)^2\beta^8}{16(1+\beta^2)^2(4+\beta^2)^2}+\frac{\alpha(1-\nu^2)\beta^4}{4\kappa^4}$$

$$\times\left[\left(\frac{9+4\beta^2}{1+\beta^2}\right)^2+\left(\frac{1+4\beta^2}{1+\beta^2}\right)^2+\left(\frac{1+4\beta^2}{4+\beta^2}\right)^2\right]$$

$$+\frac{\alpha^2}{\kappa^8}(1+4\beta^2)^2(9+4\beta^2)^2 \tag{25}$$

When $R\to\infty$, the above formula is expressed as

$$\left[\frac{\pi^4 D}{32\beta a^2 t\tau}\right]^2=\frac{16\beta^4}{81(1+4\beta^2)^2(4+\beta^2)^2}$$

$$\times\left[1+\frac{81}{25}\left(\frac{1+4\beta^2}{9+4\beta^2}\right)^2\right]\left[1+\frac{81}{25}\left(\frac{4+\beta^2}{4+9\beta^2}\right)^2\right] \tag{26}$$

$N=2$

Case I: $m+n=even$

Equating to zero the determinant of the five homogeneous equations with W_{11}, W_{13}, W_{31}, W_{33}, and W_{22} of Eqs. (16), we can obtain the eigenvalue in the same forms of Eqs. (17), (19), and (20). Here, the coefficients A_{ij}, and $F_2(\beta)$ should be replaced by

$$A_{22}=\frac{(1-\nu^2)\beta^4}{4(4+\beta^2)^2}+\frac{\alpha}{\kappa^4}(1+\beta^2)^2$$

$$A_{11}=-\frac{(1-\nu^2)\beta^4}{45(1+\beta^2)^2}+\frac{(1-\nu^2)\beta^4}{36(25+\beta^2)^2}+\frac{\alpha(9+\beta^2)^2}{144\kappa^4}$$

$$A_{13}=\frac{(1-\nu^2)\beta^4}{45(1+\beta^2)^2}+\frac{(1-\nu^2)\beta^4}{20(9+\beta^2)^2}+\frac{\alpha}{80\kappa^4}(1+\beta^2)^2$$

$$A_{31}=-\frac{81(1-\nu^2)\beta^4}{25(1+9\beta^2)^2}+\frac{81(1-\nu^2)\beta^4}{20(25+9\beta^2)^2}+\frac{81\alpha}{80\kappa^4}(1+\beta^2)^2$$

$$A_{33}=\frac{81(1-\nu^2)\beta^4}{25(1+9\beta^2)^2}+\frac{9(1-\nu^2)\beta^4}{100(1+\beta^2)^2}+\frac{9\alpha}{400\kappa^4}(1+9\beta^2)^2$$

$$F_2(\beta)=\frac{(1-\nu^2)^2\beta^8}{16}\left[\frac{1}{(1+\beta^2)^2(25+\beta^2)^2}\right.$$

$$\left.+\frac{1}{(1+\beta^2)^2(9+\beta^2)^2}+\frac{1}{(9+\beta^2)^2(25+\beta^2)^2}\right]$$

$$+\frac{\alpha(1-\nu^2)\beta^4}{64\kappa^4}\left[2+\left(\frac{9+\beta^2}{1+\beta^2}\right)^2+\left(\frac{1+\beta^2}{25+\beta^2}\right)^2\right]$$

$$+\frac{\alpha^2}{256\kappa^8}(1+\beta^2)^2(9+\beta^2)^2 \tag{27}$$

It is easily shown that Eq. (17) is reduced to Eq. (22) when $R\to\infty$.

Case II: $m+n=odd$

The expression for eigenvalue is represented again by the same equations as those for $N=1$, except that Eqs. (25) should be replaced by the following:

$$A_{12}=\frac{(1-\nu^2)\beta^4}{4(16+\beta^2)^2}+\frac{\alpha}{16\kappa^4}(4+\beta^2)^2$$

$$A_{21}=\frac{4(1-\nu^2)\beta^4}{(25+4\beta^2)^2}-\frac{16(1-\nu^2)\beta^4}{5(1+4\beta^2)^2}+\frac{\alpha}{16\kappa^4}(9+4\beta^2)^2$$

$$A_{23}=\frac{16(1-\nu^2)\beta^4}{5(1+4\beta^2)^2}+\frac{36(1-\nu^2)\beta^4}{5(9+4\beta^2)^2}+\frac{9\alpha}{80\kappa^4}(1+4\beta^2)^2$$

$$A_{32}=\frac{81(1-\nu^2)\beta^4}{4(16+9\beta^2)^2}+\frac{\alpha}{16\kappa^4}(4+9\beta^2)^2$$

$$F_1(\beta)=16(1-\nu^2)^2\beta^8\left[\frac{1}{(1+4\beta^2)^2(9+4\beta^2)^2}\right.$$

$$\left.+\frac{1}{(1+4\beta^2)^2(25+4\beta^2)^2}+\frac{1}{(9+4\beta^2)^2(25+4\beta^2)^2}\right]$$

$$+\frac{\alpha(1-\nu^2)\beta^4}{4\kappa^4}\left[2+\left(\frac{9+4\beta^2}{1+4\beta^2}\right)^2+\left(\frac{1+4\beta^2}{25+4\beta^2}\right)^2\right]$$

$$+\frac{\alpha^2}{256\kappa^8}(1+4\beta^2)^2(9+4\beta^2)^2 \tag{28}$$

When $R\to\infty$, τ_{cr} leads to Eq. (26).

In this manner, the critical shearing stresses for $N=3,4,5,\ldots$ are also obtained from Eqs. (16). The theoretical results show that the buckling shear stress is governed by the

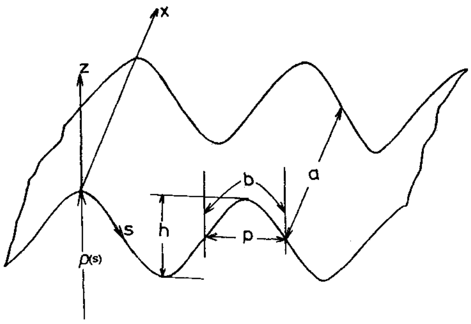

Fig. 1 Geometries and coordinate systems.

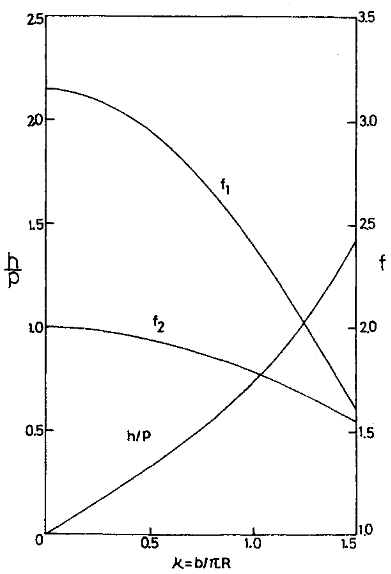

Fig. 2 Geometric relations.

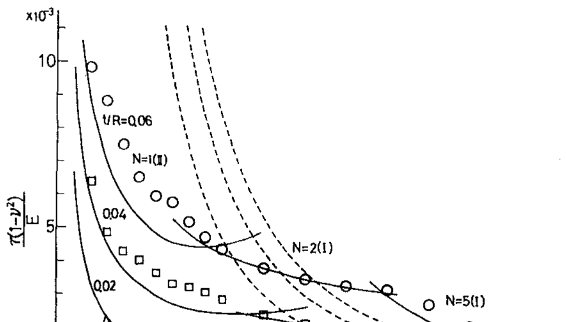

Fig. 3 Buckling shear stress of corrugated plates with $\kappa=1$.

parameters $\kappa=b/\pi R$, $\alpha=t^2/(12R^2)$, and $\beta=\ell/a$, as is the case in the buckling of corrugated plates under axial compression.[5]

Numerical Results

The values of equations such as (17) are easily obtained using a commercial hand calculator. Examples are shown in Fig. 3 for corrugated plates with $\kappa=1$ and with $t/R=0.02$, 0.04, and 0.06. Finite element results for circularly corrugated plates with the same values of κ and t/R are also plotted from Ref. 4. Both results are seen to be in good agreement. This confirms the analytical conclusion that the buckling stress is primarily governed by the parameters κ and α. It may be noticed also that virtually the same buckling stress is obtained whether the corrugation is sinusoidal or circular. The dashed curves are the results computed from orthotropic plate theory,[6,7]

$$\tau_{cr}=\frac{4C_a}{tp^2}\,\frac{4\sqrt{D_yD_x^3}}{(a/p)^2} \tag{29}$$

where

$$C_a=8.125+5.64/\theta-0.6/\theta^2$$

$$\theta=\sqrt{D_yD_x}/D_{xy}$$

$$D_x=\frac{E}{1-\nu^2}\,\frac{I}{p},\quad D_y=\frac{Et^3}{12(1-\nu^2)}\,\frac{p}{b},\quad D_{xy}=\frac{Et^3}{6(1+\nu)}\,\frac{b}{p}$$

$$I=t\int_0^b z^2\mathrm{d}s \tag{30}$$

It is seen that the orthotropic plate theory significantly overestimates the buckling strengths for short plates, whereas it gives conservative estimations for long plates.

Shear tests were performed on nine polyester circularly corrugated plates by the present authors.[4] Polyester flat rectangles were formed into circularly corrugated plates using aluminum mandrels and shear loading frames. The resulting circularly corrugated plates were thickness $t=0.188$ mm, radius $R=13.3$ mm, and half-pitch $p=26.0$ mm, which corresponds to $\kappa=0.856$ and $\alpha=16.3\times10^{-6}$. The lengths of plates were 100, 200, and 300 mm and the number of corrugations across the width of the plates was 5. Young's modulus for the material was experimentally determined to be 5.30 GPa and Poisson's ratio was assumed to be equal to 0.3. The shear load was applied to the specimen through the beam shear fixture by means of a hand screw jack. The test beam was vertically cantilevered to a heavy steel table and loaded at the top with a horizontal applied load. The comparison of the present analysis with the experimental results is shown in Fig. 4. It is seen that theoretical and experimental results are in good agreement.

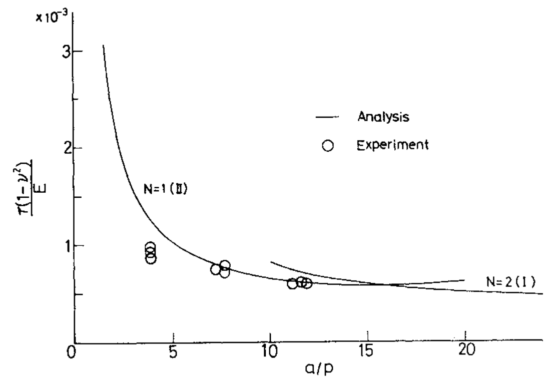

Fig. 4 Comparison of analytical and experimental results.

Conclusions

The buckling of a sinusoidally corrugated plate in shear has been analytically examined. The conclusions of the present study are summarized as follows:

1) The governing equation has been derived in terms of only the lateral displacement component W_{mn}. The eigenvalue for the equation has been obtained explicitly in closed form, so that shear buckling stress is easily and accurately predicted.

2) The solution shows that the most significant parameters of the cross section governing the buckling stress are $\kappa = b/\pi R$ and $\alpha = t^2/(12R^2)$.

3) The present analytical results for sinusoidally corrugated plates are in good agreement with both finite element and experimental results for circularly corrugated plates having the same values of κ and α as those of sinusoidal corrugation.

4) Orthotropic plate theory can overestimate the buckling strength for a short plate. On the other hand, it gives conservative prediction for a long plate.

Acknowledgment

This paper was originally presented at the XVIth International Congress of Theoretical and Applied Mechanics, Lyngby, Denmark, Aug. 19-25, 1984. After the conference, the first author received a copy of Ref. 8 from Prof. C. Libove of Syracuse University. The authors extend their gratitude to Prof. Libove for his useful comments on this paper.

References

[1]Libove, C., "Buckling of Corrugated Plates in Shear," *Stability of Structures under Static and Dynamic Loads*, ASCE, New York, 1977, pp. 453-462.

[2]Perel, D. and Libove, C., "Elastic Buckling of Infinitely Long Trapezoidally Corrugated Plates in Shear," *Transactions of ASME*, Ser. E, Vol. 45, No. 3, 1978, pp. 579-582.

[3]Sanbongi, S., "Elastic Buckling Analysis of Corrugated Plates under Uniform Compression," National Aerospace Laboratory, Tokyo, Rept. TR-604, 1980 (in Japanese).

[4]Sanbongi, S. and Toda, S., "Buckling Strength of Corrugated Shear Webs," National Aerospace Laboratory, Tokyo, Rept. TR-759, 1983 (in Japanese).

[5]Toda, S., "Buckling of Sinusoidally Corrugated Plates under Axial Compression," *AIAA Journal*, Vol. 21, Aug. 1983, pp. 1211-1213.

[6]Timoshenko, S. P. and Gere, J. M., *Theory of Elastic Stability*, 2nd ed., McGraw-Hill Book Co., New York, 1961, pp. 379-385, 405-408.

[7]Japan, C. R. C. (ed.), *Handbook of Structural Stability*, Corona Publishing Co., Tokyo, 1971.

[8]Hussain, M. I. and Libove, C., "Shear Buckling of Corrugated Plates with Quasi-Sinusoidal Corrugations," Dept. of Mechanical and Aerospace Engineering, Syracuse University, Syracuse, NY, Rept. MAE-5293-T4, July 1977.